Communication Networks:
Technology & Protocols

Stavros Tripakis (stavros@eecs)

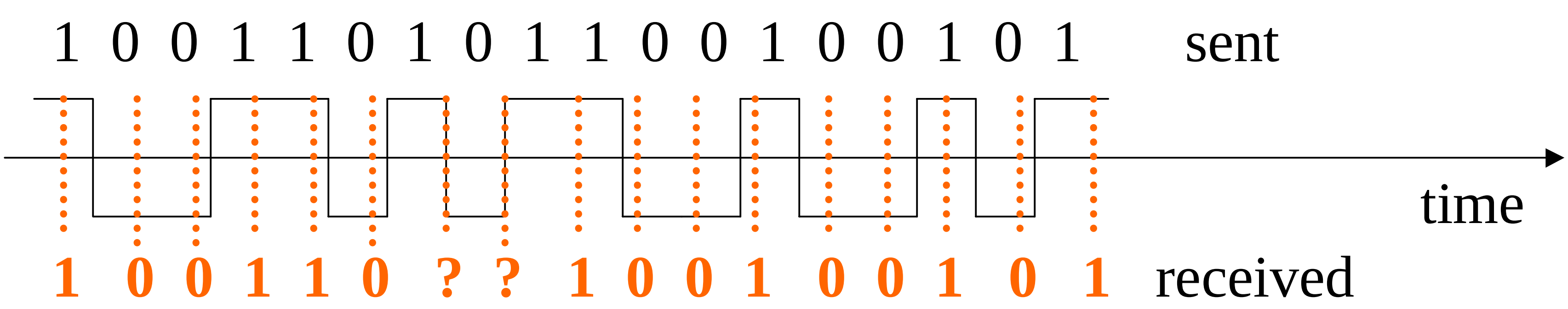
Lecture 24
October 20

Error detection & correction

- Detect (and possibly correct) errors in transmission introduced by physical layer.
- Error detection happens at different layers:
  - At data-link: **bit-error** detection/correction: check whether all bits in a packet are received correctly. If yes, deliver packet to higher layer; otherwise, discard packet (or, if possible, correct it and deliver it).
  - At network or transport layers: **packet-error** detection: check whether packet is correct. If yes, deliver to higher layer, otherwise, discard.

Where are bit-errors and where do they come from ?

- Bit inversion, e.g., send 100, receive 101.
- Lost/added bits, e.g., send 100, receive 1000.
- Causes of errors:
  - Noise: send signal of -5 Volts, receive signal of +1 Volts.
  - De-synchronization of sender/receiver clocks:



  - In wireless channels: multipath, shadowing, etc.

Error-detection/correction codes

- **Block** codes: memory-less codes.
  - For each "block" of (data) bits M, add some redundant bits R = f(M), and transmit MR (codeword).
- **Convolutional** codes: codes with memory.
  - R = f(M, s), where s is the state of the code (depends on previous blocks).
- Criteria for choosing a code:
  - How much redundancy (overhead) does it add ?
  - How many errors can it detect/correct ?
  - How expensive is it to compute ?

Simple solution: send multiple copies

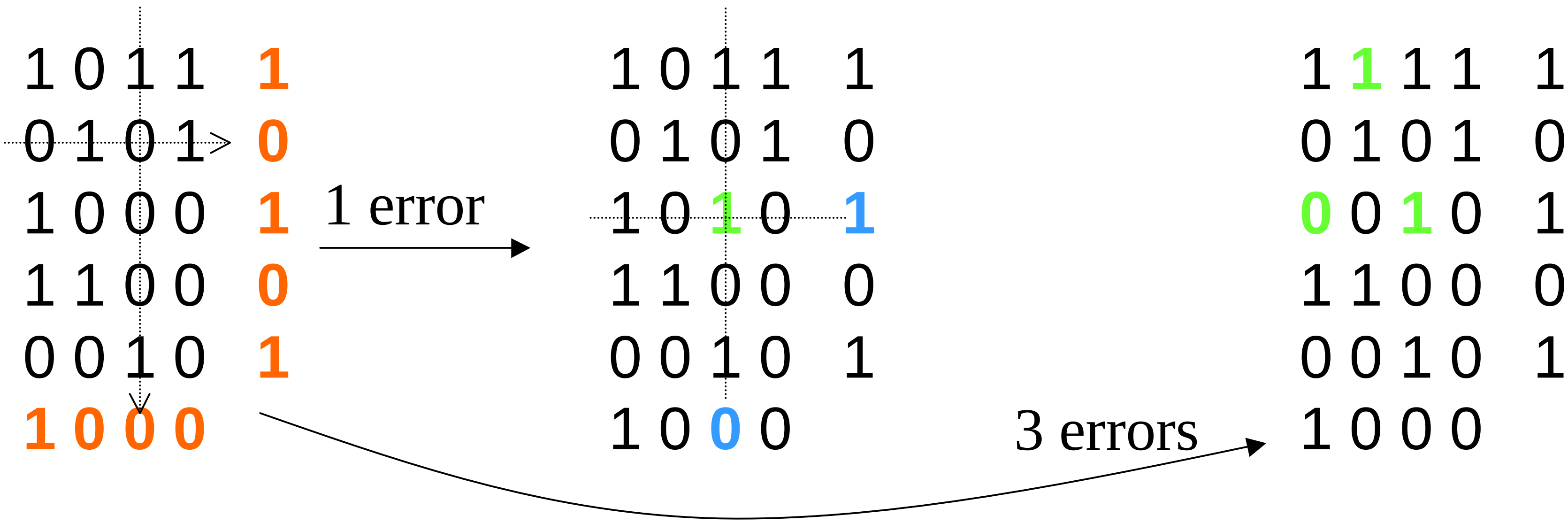
- Each packet is sent k times, $k>1$.
- Error detection (not all copies the same).
- Error correction (majority rule).
- Reduces PER: packet error rate.
- Too much redundancy: channel capacity divided by k.
- Can do better using more clever codes.

Parity codes

- Simple parity:
  - For each k bits, add 1 parity bit:
    - Even/Odd parity: some of 1s must be even/odd.
    - E.g.: 1001 --- even parity ---> 1001**0**
  - Detects an odd number of bit-errors, e.g.:
    - 1001 ---> 10010 --- channel ---> 10110 (1 error) (receiver knows there is an error).
    - 1001 ---> 10010 --- channel ---> 01010 (2 errors) (receiver detects no error).

Parity codes

- Two-dimensional parity:

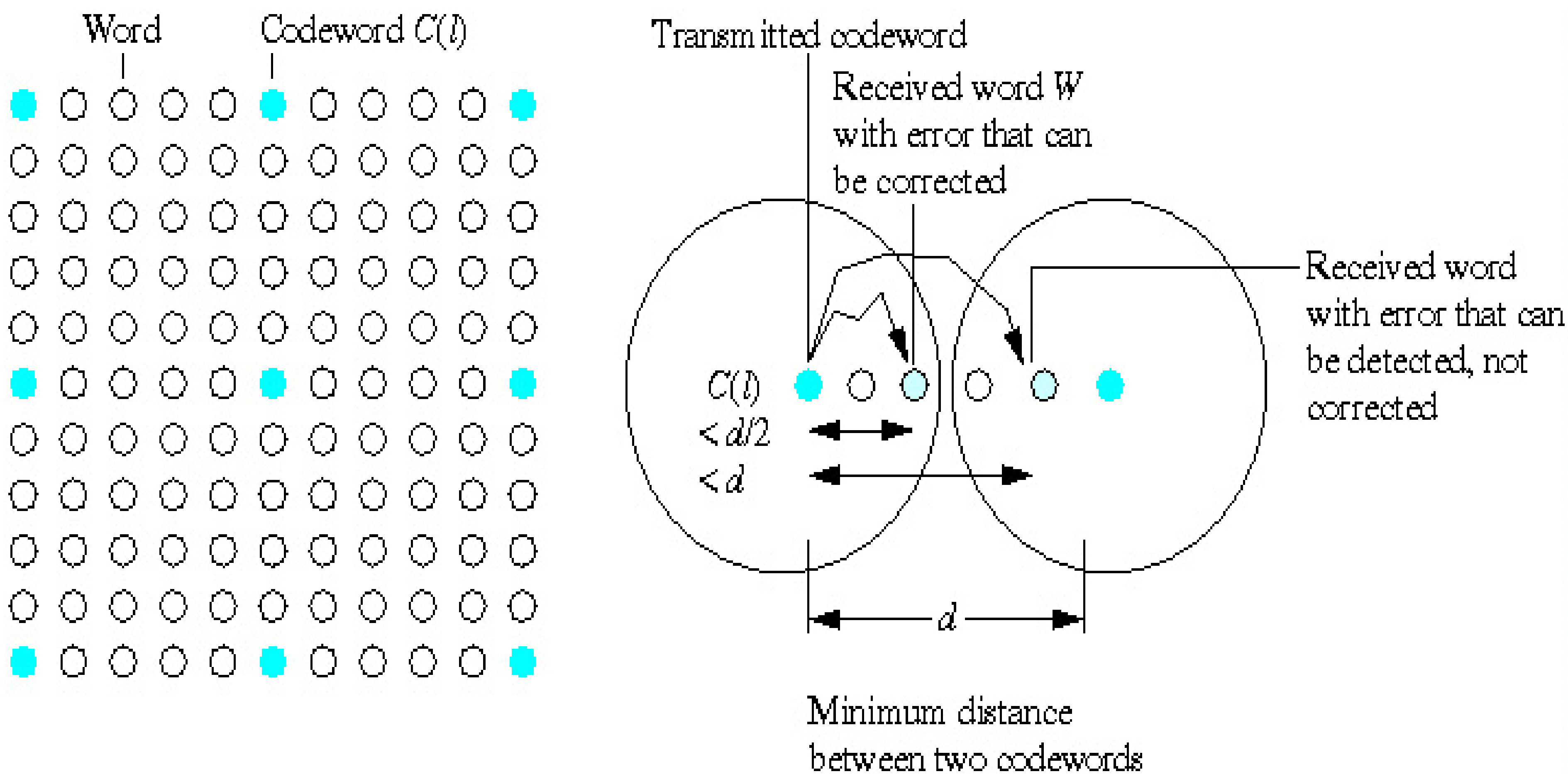


  - Detection of 1,2,3-bit errors.
  - Correction of 1 bit-error per row/column.

Hamming codes

- Bit-words of length *n* : points in n-dimensional boolean space.
- *Hamming distance* between two words: the number of bits in which the words differ, e.g. |1001, 1000| = 1, |1001, 1100| = 2.
- If valid codewords have minimum distance d, then:
  - If less than d bit-errors per block, error detection.
  - If less than d/2 bit-errors per block, also correction.

Hamming codes



Hamming codes

- What is the minimum amount of redundancy ?
- Hamming: can always correct 1 bit-error per codeword of length n, with log2(n+1) redundant bits.
- Idea: if codeword is *b(1) b(2) … b(n),* then:
  - b(1), b(2), b(4), b(8), …, b(2^k) : parity bits.
  - All other bits : data bits.
  - Parity bit b(2^i) checks the parity of all bits b(j), s.t. j written in base-2 has 1 in position i.
  - With k data bits, it is called an (n,k) code.

Hamming codes: example

- Example: data = 0010. Find the (7,4) codeword.
  - There are 7-4=3 parity bits.
  - The codeword is: b1 b2 b3 b4 b5 b6 b7, where b3=0, b5=0, b6=1, b7=0.
  - Parity bit b1 checks b3 (011), b5 (101), b7 (111). Therefore, b1 = 0 (assuming even parity).
  - Parity bit b2 checks b3 (011), b6 (110), b7 (111). Therefore, b2 = 1.
  - Parity bit b4 checks b5 (101), b6 (110), b7 (111). Therefore, b4 = 1.
- Final codeword: 0101010.

Hamming codes: example (cont'd)

- Final codeword: 0101010.
- Channel corrupts codeword: 0101110.
- Receiver can tell which bit is inverted:
  - b1 = 0 is the parity bit of 0, 1, 0 : wrong.
  - b2 = 1 is the parity bit of 0, 1, 0 : correct.
  - b4 = 1 is the parity bit of 1, 1, 0 : wrong.
  - The inverted bit is the intersection of the bits checked by b1 and those checked by b4, that is, {b3,b5,b7} and {b5,b6,b7} = {b5,b7}.
  - The inverted bit is not b7, since this is checked by b2 (so, b2 would be wrong, but it is correct).

Cyclic redundancy check (CRC)

- Idea:
  - There is a generator G, known to both sender and receiver.
  - If M is the data to be sent, sender transmits codeword (M, M mod G).
  - Receiver gets (X, Y) and checks whether

    X mod G = Y.

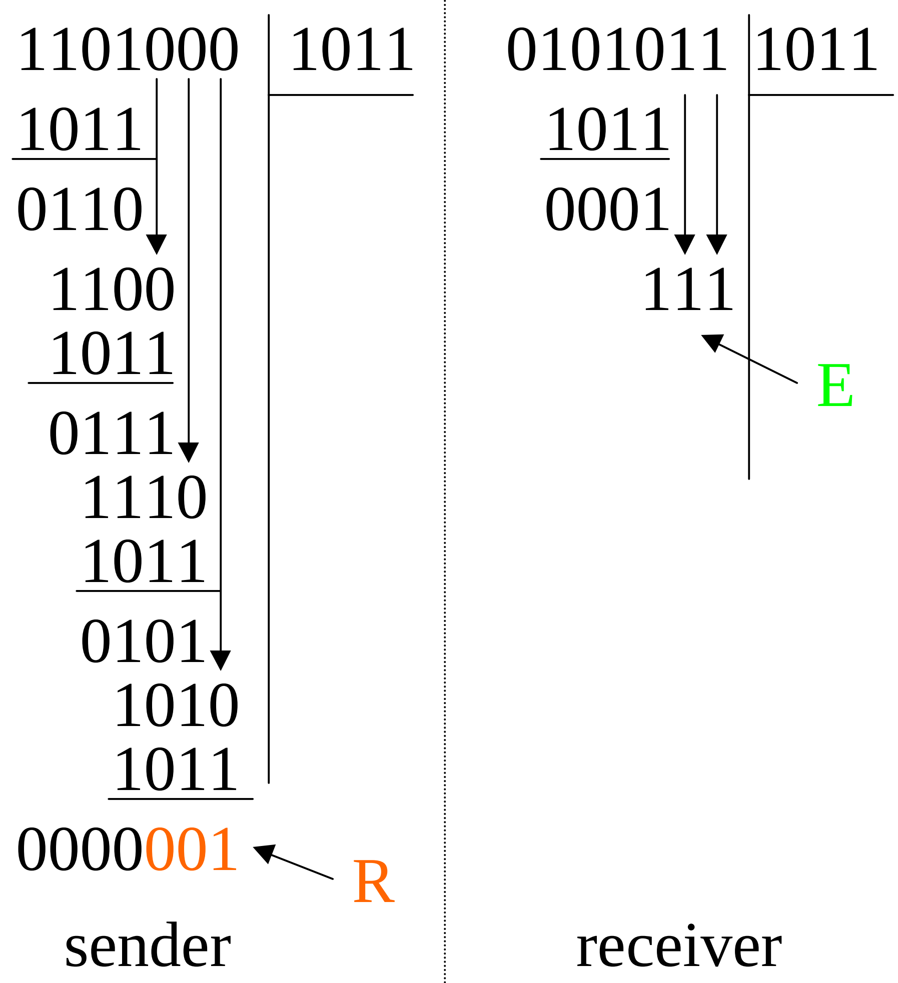
    If yes, OK, otherwise discard packet.
  - If G is properly chosen, then if errors occur, it should be improbable that they result in such X and Y that X mod G = Y.

Cyclic redundancy check (CRC)

- In reality:
  - M, G, etc, are base-2 numbers (bit-strings).
  - Operations are done modulo 2 without carry.
    E.g., 1+1 = 0+0 = 0, 1+0 = 0+1 = 1.
  - There are r redundancy bits.
  - Sender transmits T = M x 2^r + R, where:
    R = (M x 2^r) mod G (remainder).
  - Receiver gets T' (possibly corrupted) and checks whether E = T' mod G = 0. If yes, it assumes no error.
  - In some codes (error-correcting codes or ECC), the 1s in E tell where errors occurred.

CRC : example

- Let M=1101, G=1011, r=3.
- Sender computes R:
  long division of 1101000
  by 1011.
- Sender transmits
  1101000+R = 1101001 = T.
- Suppose channel corrupts
  string: 0101011 = T'.
- Receiver computes
  E = T' mod G = 111.
- E not 0, so an error occurred.



CRC : properties

- CRC based on the theory of finite fields.
- Bit-words can be seen as polynomials:
  e.g., 1001 : $x^3 + 1$, $x^4 + x^2 + 2x$: 10110
- It can be shown that CRC can detect :
  - All 1-bit errors.
  - Almost all 2-bit errors, if G(x) has a factor with at least three terms, e.g., $G(x) = (x+1)(x^2+x+1) = x^3 + 2x^2 + 2x + 1$: 1111.
  - Any odd number of errors, if G(x) has a factor x+1.
  - All bursts of up to m errors, if G(x) is of degree m.
  - Longer burst errors with probability $1-2^{-m}$, if bursts are randomly distributed.

CRC : properties

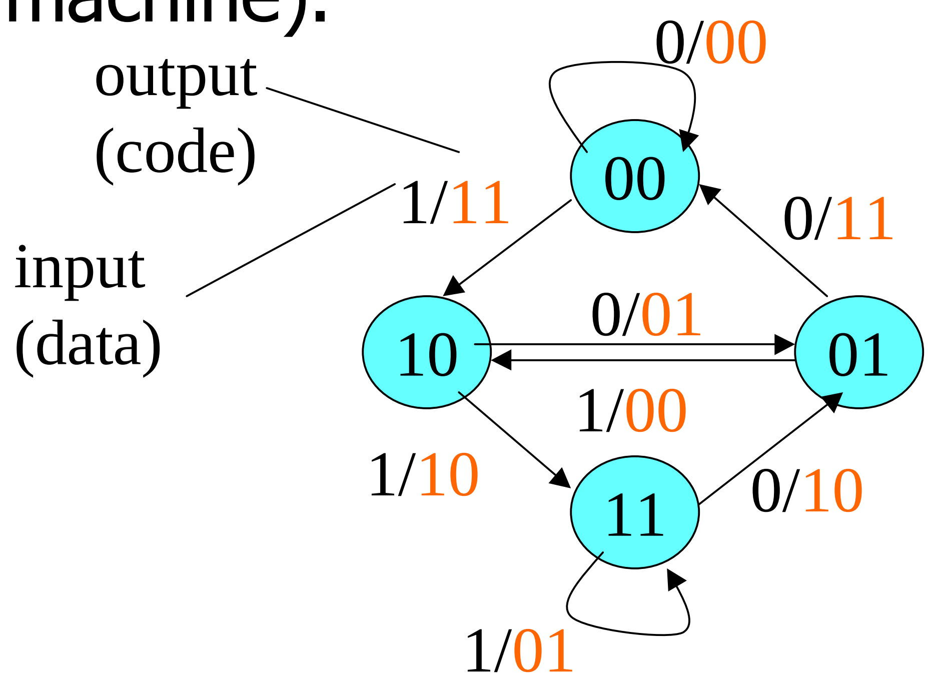
- Generators are standardized:
  - CRC-m means a generator polynomial of degree m, that is, a bit-word of length m+1.
  - CRC-8 : 100000111 (or x^8 + x^2 + x + 1).
  - CRC-10: 11000001111.
  - CRC-12: 1100000001101.
  - CRC-16, CRC-32, CRC-CCITT.
- Ethernet, FDDI use CRC-32, ATM uses CRC-8 and CRC-10 (bit-error rates in fibers are extremely small, e.g., 10^-14).
- Implemented in hardware using a shift register of r bits and XOR gates (addition/subtraction modulo 2 w/o carry is XOR).

Other block codes

- Permutation codes: aimed at detecting error **bursts**. E.g., *interleaved* Hamming codes:
  - m consecutive codewords (each of length n) are written in an m x n matrix.
  - Matrix is transmitted column-wise instead of row-wise.
  - A burst of up to m bit-errors will appear as 1-bit error per m-bit column.
- Turbo codes: permutations + multiple encoders/decoders that "cooperate".

Convolutional codes

- Codes with memory (e.g., represented by an FSM, a finite-state machine).
- Encoder:



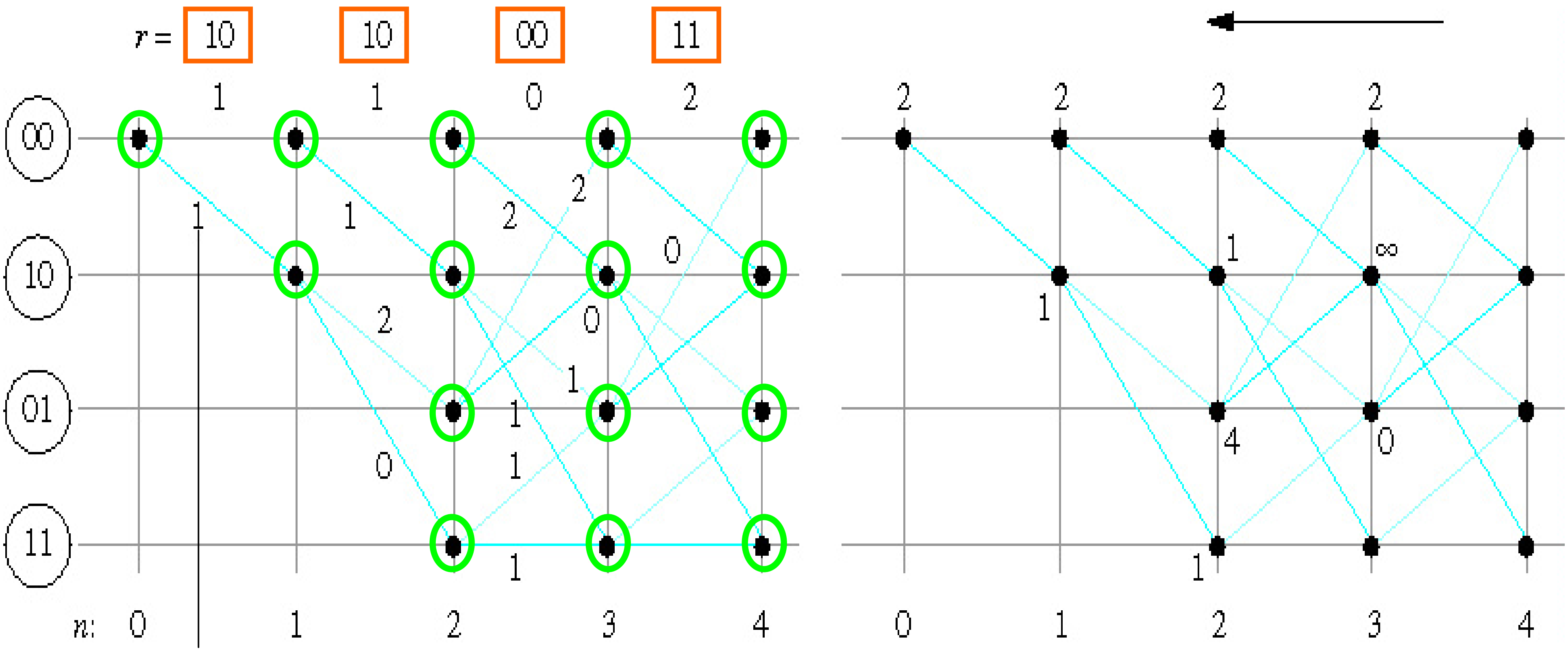
E.g.: Input = 010111
Output = 001101001001

  - Code *rate* : 1/2 (2 output bits per each input bit).
  - Implemented as a circuit.

Convolutional codes

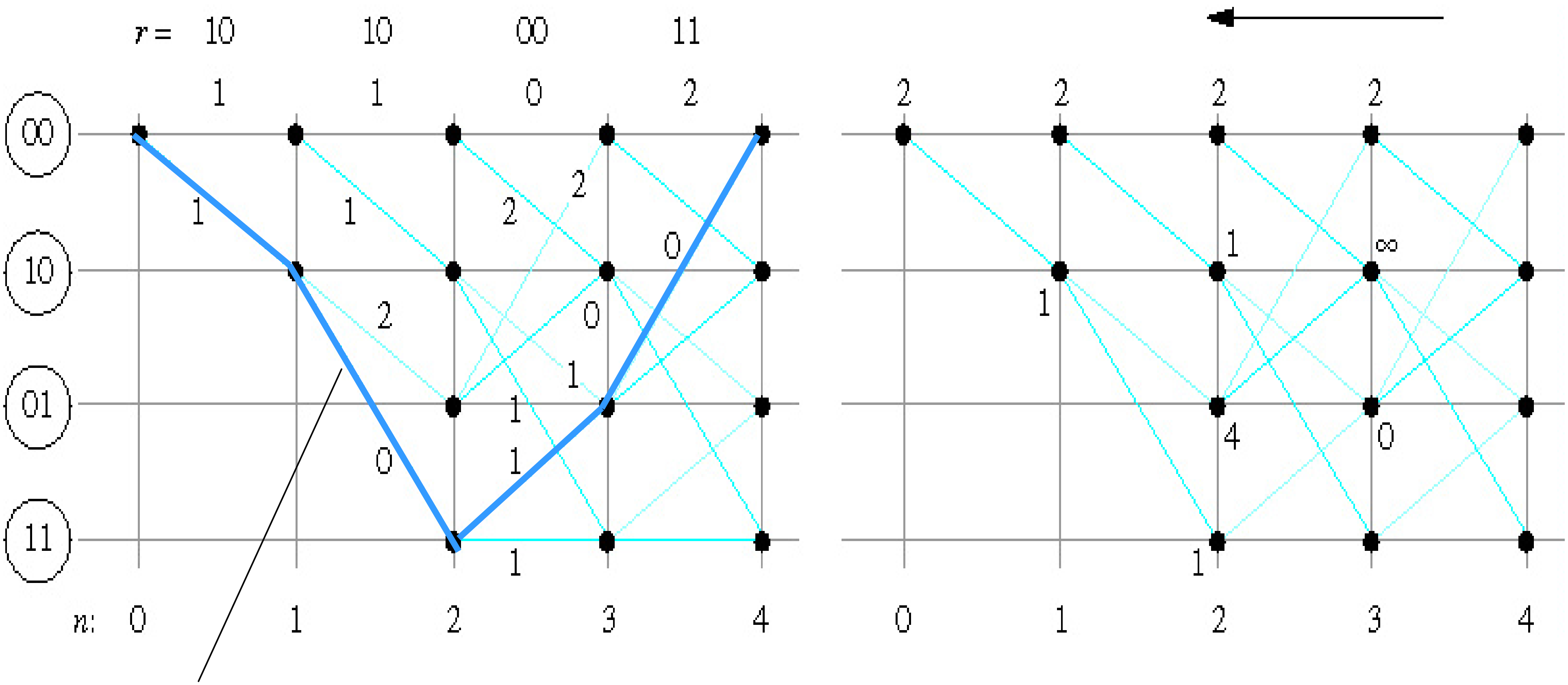
- Decoder:
  - Knows the FSM of the encoder.
  - Keeps track of the possible states the encoder might be in, the likelyhood of each state and the history to get to that state.
  - Upon receiving a bit-word, updates its information according to the current states/likelyhoods, and the FSM. E.g., it knows that if encoder was at state 00, then it couldn't have sent 01, so an error must have occured.
  - At the end, it chooses the most-probable history.
  - Implementation: *pruning* is used to remove useless histories and keep complexity low.

Convolutional codes



Penalty : how many bits would have to be corrupted, for this step to be possible.

Convolutional codes



At the end, choose the path with the smallest total penalty.

Internet checksum

- Used in IP, TCP, UDP.
- Idea:
  - Sender adds up data by 32-bit words and puts the result into the checksum field.
  - Receiver adds up data and compares result to the checksum: if different, data (or checksum) must have been corrupted.
- Weaker detection properties.
- Easy to implement in software.
- Adequate, since almost all errors caught already at data link layer.